SLC (University of Delhi)
Shyam Lal College

Saraswati IKS Centre

Project Title: An Action based Study to Explore & Connect Select IKS in Contemporary Discourses & Practices

Reviving Vedic Mathematics for Modern Calculus Applications

Intern Name

Bhaskar Gupta, B.Sc. Physical Science with Chemistry,
Shyam Lal College

Mentor Name

Dr. Seema Guglani, Assistant Professor,Department of Mathematics,
Shyam Lal College

1. INTRODUCTION

Calculus, a fundamental mathematical technique for modelling and analysing continuous change, occupies a significant realm within the broader field of mathematics, commonly known as infinitesimal calculus. Traditionally attributed to the 17th-century discoveries by Isaac Newton and Leibniz, the origins of calculus can be traced back to Siddhantasiromani of Bhaskara II, who described key principles as early as 1150 AD.

In the subsequent centuries, the Kerala School of Astronomy and Mathematics emerged as a hub for the advancement of calculus concepts within the framework of Indian mathematics (Williams, 2005). Pioneering mathematicians like Madhava of Sangamagrama and their successors contributed substantially, introducing transformative ideas such as the Taylor series and infinite series approximations. The foundational components of calculus, Integration, and Differentiation, began to take shape.

Parallel to these historical developments, there exists an intriguing discourse on Vedic Mathematics. Sri Bharati Krishna Tirthaji (1884–1960), an Indian mathematician, claimed to rediscover Vedic Mathematics from the Vedas between 1911 and 1918. Tirthaji asserted that Vedic Mathematics comprises sixteen (16) sutras, or formulae, and thirteen (13) sub-sutras, offering solutions to diverse mathematical problems, including algebra, geometry, arithmetic, and, notably, calculus (Sanjeev, 2018). This paper will explore the efficiency of Vedic Mathematics in understanding concepts of calculus in differentiation and integration.

2. DIFFERENTIAL CALCULUS

The calculation of how one parameter varies concerning another involves differential equations and differential calculus (Kumar, 2021). By drawing an angle at a particular point and calculating the slope, one can geometrically determine the differential of any function at that position. For a function y = f(x):

$$\frac{dy}{dx} = D_1 = f'(x)$$

$$\frac{d^2y}{dx^2} = D_2 = f''(x)$$

This section explores the application of Vedic Mathematics Sutras in differential calculus (Wlliams, 2005).

2.1. Urdhva Triyagbhyam Sutra

Successive differentiation can be efficiently carried out by the use of Urdhva Triyagbhyam sutra Its English meaning is vertically and crosswire.

2.1.1. Derivative of product of two functions

Vedic Method

Example: To find $\left(\frac{d}{dy}\right)(y^4 Siny)$ or $D_1(y^4 Siny)$

Step 1: Write the two parts one below the other.

p y^4

q $Sin(y)$

Step 2: Differentiate each term once and write next to them.

	Function	Differentiation
p	y^4	$4y$
q	$Sin(y)$	$Cos(y)$

Step 3: Cross multiply each term and add for the result.

p	$1(y^4)$	$1(4y^3)$
q	$Sin(y)$	$Cos(y)$

Result: $y^4Cos(y) + 4y^3Sin(y)$

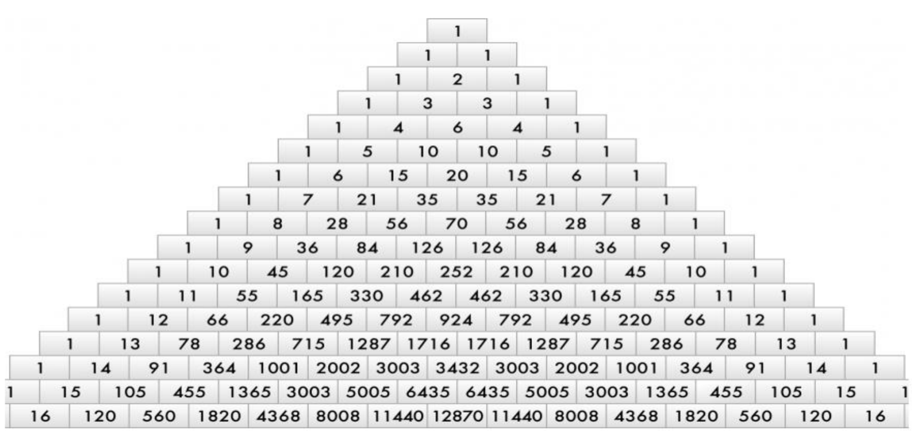
For second derivative write the first and second derivatives and multiply vertically and crosswire.

Example: To find $D_2(y^4Siny)$

	Function	First Derivative	Second Derivative
p	$1(y^4)$	$2(4y^3)$	$1(12y^2)$
q	$Sin(y)$	$Cos(y)$	$-Sin(y)$

Result: $-y^4Sin(y) + 12y^2Sin(y) + 8y^3Cos(y)$

The extra factor 2 in the middle term comes from the Meru Prastara of Pingala.



Example: To find $D_3(y^4 Siny)$

	Function	First Derivative	Second Derivative	Third Derivative
p	(y^4)	$(4y^3)$	$(12y^2)$	$(24y)$
q	$Sin(y)$	$Cos(y)$	$-Sin(y)$	$-Cos(y)$

Result: $-\mathrm{y^4Cos(y)} + 24\mathrm{ySin(y)} - 12\mathrm{y^3Sin(y)} + 36\mathrm{y^2Cos(y)}$

The four products are multiplied by 1,3,3,1 resp.

Conventional Method of finding 1st, 2nd and 3rd derivatives of product.

1st Derivative:

$$\left(\frac{d}{dy}\right)(y^4 Sin(y))$$

p= y^4, q= $Sin(y)$

Formula: $\left(\frac{d}{dy}\right)(p.q) = p\left(\frac{d}{dx}q\right) + q\left(\frac{d}{dx}p\right)$

Solving for: $\left(\frac{d}{dy}\right)(y^4 Sin(y))$

$$= y^4\left(\frac{d}{dy}Sin(y)\right) + Sin(y)\left(\frac{d}{dy}y^4\right)$$

$$= y^4 Cos(y) + 4y^3 Sin(y)$$

2nd Derivative:

$$(\frac{d}{dy})(y^4 Cos(y) + 4y^3 Sin(y)$$

$$= y^4\left(\frac{d}{dy}Cos(y)\right) + Cos(y)\left(\frac{d}{dy}y^4\right) + 4y^3\left(\frac{d}{dy}(\mathrm{Sin}\,y)\right) + Sin(y)\left(\frac{d}{dy}4y^3\right)$$

$$= y^4(-Sin(y)) + 4y^3Cos(y) + 4y^3Cos(y) + 12y^2Sin(y)$$

$$= -y^4Sin(y) + 8y^3Cos(y) + 12y^2Sin(y)$$

3rd Derivative:

$$\left(\frac{d}{dy}\right)\left(-y^4Sin(y) + 8y^3Cos(y) + 12y^2Sin(y)\right)$$

$$= -y^4\left(\frac{d}{dy}Sin(y)\right) + Sin(y)(\frac{d}{dy}(-y^4) + 8y^3\left(\frac{d}{dy}Cos(y)\right) + Cos(y)\left(\frac{d}{dy}8y^3\right) + 12y^2\left(\frac{d}{dy}Sin(y)\right) + Sin(y)\left(\frac{d}{dy}12y^2\right)$$

$$= -y^4Cos(y) - 4y^3Sin(y) - 8y^3Sin(y) + 24y^2Cos(y) + 12y^2Cos(y) + 24ySin(y)$$

$$= -y^4 Cos(y) - 12y^3 Sin(y) + 36y^2 Cos(y) + 24y Sin(y)$$

Conventional method of Leibnitz Rule for determining n^{th} order differential of product of two functions:

Consider functions p and q, both dependent on x, with derivatives up to the nth order existing

$f^{(n)}(x) = (pq)^n = {}^nC_0 pq^{(n)} + {}^nC_1 p' q^{(n-1)} + {}^nC_2 p'' q^{(n-2)} + {}^nC_3 p''' q^{(n-3)} + \ldots\ldots\ldots + {}^nC_{n-1} p^{n-1} q' + p^n q$ [A]

Example: Evaluate n^{th} order differential of $y=x^4.e^x$

Let $p= x^4$; $p' = 4x^3$; $p'' = 12x^2$; $p''' = 24x$; $p'''' = 24$; $p''''' = 0$

And $q= e^{3x}$; $q' = e^{3x}$; $q'' = e^{3x}$; $\ldots\ldots\ldots$ $q^n = e^{3x}$

Here, $p''''' = 0$

$\therefore$ Leibnitz rule can be applied till 4^{th} power of p.

$\therefore f^{(n)}(x) = (pq)^n$

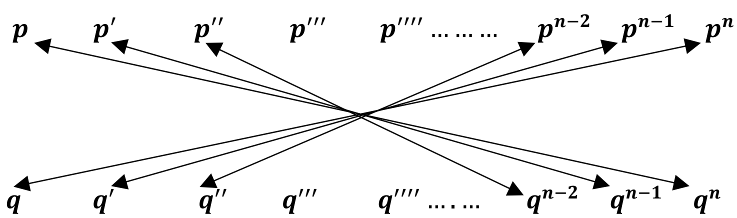
$= {}^nC_0 pq^{(n)} + {}^nC_1 p' q^{(n-1)} + {}^nC_2 p'' q^{(n-2)} + {}^nC_3 p''' q^{(n-3)} + {}^nC_4 p'''' q^{(n-4)}$

$= x^4 e^{3x} + 4nx^3 e^{3x} + (12/2!)\ n\times(n-1)\ x^2 e^{3x} + (24/3!) \times n\times(n-1) \times(n-2)\ xe^{3x} + (24/4!) \times n\times (n-1)(n-2)(n-3)\ e^{3x}$

$= x^4 e^{3x} + 4nx^3 e^{3x} + 6n\times(n-1)\ x^2 e^{3x} + 4n\times(n-1)\times(n-2)\ xe^{3x} + n\times(n-1)\times(n-2)\times(n-3)\ e^{3x}$

$\therefore f^{(n)}(x) = e^{3x}\,[x^4 + 4nx^3 + 6n\times(n-1)\ x^2 + 4n\times(n-1)\times(n-2)\ x + n\times(n-1)\times(n-2)\times(n-3)]$

Using Urdhva Triyagbhyam Sutra and Binomial expansion for n^{th} order,



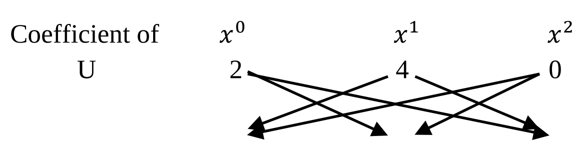
$f^{(n)}(x) = (pq)^n = {}^nC_0 pq^{(n)} + {}^nC_1 p' q^{(n-1)} + {}^nC_2 p'' q^{(n-2)} + {}^nC_3 p''' q^{(n-3)} + \ldots\ldots\ldots {}^nC_{n-2} p^{(n-2)} q'' + {}^nC_{n-1} p^{(n-1)} q' + p^{(n)} q$ [B]

The equation above, denoted as [A], is equivalent to the result [B] obtained through the Vedic method along with binomial expansion, condensing the solution into a single step.

2.1.2. Derivative of division of polynomials

Vedic Technique to find derivative of division of polynomial functions.

Example: Differentiate $y = \frac{2+4x}{2x+2x^2}$



V 0 2 2

$[(0 \times 4) - (2 \times 2)](1 - 0)$ $[(0 \times 2) - (4 \times 2)](2 - 1)$

$[(0 \times 0) - (2 \times 2)](2 - 0)$

Let $y = \frac{2+4x}{2x+2x^2} = \frac{2x^0+4x^1+0x^2}{0x^0+2x^1+2x^2}$

$$\frac{dy}{dx} = \frac{[(0\times4)-(2\times2)](1-0)+ [(0\times0)-(2\times2)](2-0)x+ [(0\times2)-(4\times2)](2-1)x^2}{(2x+2x^2)^2}$$

$$= \frac{(-4) + (-4)2x + (-8)x^2}{(2x + 2x^2)^2}$$

$$= \frac{-4 - 8x - 8x^2}{(2x + 2x^2)^2}$$

Conventional Method

$$\frac{dy}{dx} = \frac{(2x + 2x^2)4 - (2 + 4x)(2 + 4x)}{(2x + 2x^2)^2}$$

$$= \frac{(8x+8x^2)-(4+16x+16x^2)}{(2x+2x^2)^2}$$

$$= \frac{(8x+8x^2-4-16x-16x^2)}{(2x+2x^2)^2}$$

$$\therefore \frac{dy}{dx} = \frac{-4-8x-8x^2}{(2x+2x^2)^2}$$

Example: Differentiate $y = \frac{1+4x+9x^2+3x^3}{2+3x+4x^2+9x^3}$

Vedic Method

$$\therefore \frac{dy}{dx} = \frac{\left\{\begin{matrix} (2\times4-1\times3)(1-0)+(2\times9-1\times4)(2-0)x+(2\times3-1\times9)(3-0)x^2+ \\ (3\times9-4\times4)(2-1)x^2+(3\times3-4\times9)(3-1)x^3+(3\times4-9\times9)(3-2)x^4 \end{matrix}\right\}}{(2+3x+4x^2+9x^3)^2}$$

$$= \frac{\left\{\begin{matrix} (8-3)(1)+(18-4)(2)x+(6-9)(3)x^2+ \\ (27-16)(2)x^2+(9-36)(3)x^3+(12-81)(1)x^4 \end{matrix}\right\}}{(2+3x+4x^2+9x^3)^2}$$

$$= \frac{5+28x-9x^2+11x^2-54x^3-69x^4}{(2+3x+4x^2+9x^3)^2}$$

$$= \frac{5+28x+2x^2-54x^3-69x^4}{(2+3x+4x^2+9x^3)^2}$$

Conventional Method

$$\frac{dy}{dx} = \frac{(2+3x+4x^2+9x^3)(4+18x+27x^2)-(1+4x+9x^2+27x^2)(3+8x+27x^2)}{(2+3x+4x^2+9x^3)^2}$$

$$= \frac{5+28x+2x^2-54x^3-69x^4}{(2+3x+4x^2+9x^3)^2}$$

2.2. Calana- Kalanābhyām Sūtra

First derivative and the square root of discriminant of a quadratic equation can related with the help of Calana-Kalanābhyām Sutra. Commonly referred to as the Calculus Formula for evaluating the two roots of a quadratic equation of degree 2, as named by Shri Bharathi Krishna, this sutra asserts that the first derivative of the quadratic equation is equal to the square root of its discriminant.

i.e., $D_1 = \pm\sqrt{Discriminant} = \pm\sqrt{b^2 - 4ac}$

Thus, by simplifying two equations we get roots of the quadratic equation.

Example: Evaluate ($y^2 - 1y - 12 = 0$)

First differential: $D_1 = 2y - 1$

$\pm\sqrt{b^2 - 4ac} = \pm(\sqrt{1 + 48}) = \pm(\sqrt{49}) = \pm(7)$

As per the Sutra $D_1 = \pm\sqrt{Discriminant}$

$2y - 1 = \pm(7)$

So, $y = 4 \; or \; y = (-3)$

2.3. Gunaka Samuccaya Sutra

If the factors of a polynomial are known, its derivative can be calculated using the Gunaka Samuccaya sutra, which highlights the relationship between factors and their successive differentials.

Rule for finding differentials involves first expressing the polynomial in standard form with 1 as the constant term preceding the highest-degree term. Then, the polynomial is broken down into linear factors. According to the Guakasamuccaya Sutra, if a polynomial is the product of factors, adding those factors yields the first differential, denoted as D1. The procedure for finding differentials for quadratic, cubic, and quartic expressions, as well as polynomials with a fifth power, along with examples, is elucidated below.

2.3.1. Differential of quadratic expression

As per the Guakasamuccaya Sūtra, if a quadratic equation can be factored into two linear factors such as t_1= w+m and t_2= w+n, then adding these two linear factors yields the first differential, denoted as D1.

D_1= 1! [Sum of product of factor taken (2-1) = 1 at a time]

$D_1 = t_1 + t_2 =$ (w+m)+(w+n)

Second differential D_2= 2!

Example: Find differential of $w^2 - 9w + 14$

As per current method,

$D_1 = \left(\frac{d}{dw}\right)(w^2 - 9w + 14) = 2w - 9$

$D_2 = \left(\frac{d}{dw}\right)(2w - 9) = 2 = 2$

Suppose U = $w^2 - 9w + 14$,

U= (w-2)(w-7)

According to Guakasamuccaya Sūtra,

D_1 = $(w-2)+(w-7) = 2w - 9$

D_2= 2!

2.3.2. Differential of Cubic Expression

Cubic expressions involves three linear factors

t_1= (w+r) , t_2= (w+s) , t_3=(w+t)

D_1= 1! [Sum of factor product taken (3-1)= 2 taken at a time]

D_1= [$t_1t_2 + t_2t_3 + t_1t_3$]

= [(w+r)(w+s)+(w+s)(w+t)+(w+r)(w+t)]

D_2= 2![Sum of factor product taken (3-2)=1 at a time]

D_2= 2![$t_1 + t_2 + t_3$] = 2![(w+r)+(w+s)+(w+t)]

Third Differential D_3= 3!

Example: Find differentials of $w^3 - 6w^2 + 5w + 12$

Let E= $w^3 - 6w^2 + 5w + 12 = (w+1)(w-3)(w-4)$

D_1= [(w+1)(w-3)]+[(w-3)(w-4)]+[(w-4)(w+1)]

= $w^2 - 2w - 3 + w^2 - 7w + 12 + w^2 - 3w - 4$

= $3w^2 - 12w + 5$

D_2= 2![(w+1)+(w-3)+(w-4)]

= $6w - 12$

D_3= 3! = 6

2.3.3. Differential of bi-quadratic expression:

Bi quadratic expression involves four linear factors

t_1= (w+m) , t_2= (w+n) , t_3= (w+p) , t_4= (w+q)

D_1= 1! [Sum of factor product taken (4-1)=3 at a time]

D_1= $t_1t_2t_3 + t_1t_2t_4 + t_2t_3t_4 + t_1t_3t_4$

$= (w+m)(w+n)(w+p) + (w+m)(w+n)(w+q) + (w+n)(w+p)(w+q)+(w+m)(w+p)(w+q)$

D_2= 2! [Sum of factor product taken (4-2)=2 at a time]

$D_2 = 2!\ [t_1t_2 + t_1t_3 + t_1t_4 + t_2t_3 + t_2t_4 + t_3t_4]$

$= 2!\ [(w+m)(w+n) + (w+m)(w+p) + (w+m)(w+q) + (w+n)(w+p) + (w+n)(w+q) + (w+p)(w+q)]$

D_3= 3! [Sum of factor product taken (4-3)=1 at a time]

$D_3 = 3!\ [t_1 + t_2 + t_3 + t_4]$

$= 3!\ [(w+m) + (w+n) + (w+p) + (w+q)]$

Fourth Differential $D_4 = 4!$

Example: Find differentials of bi-quadratic equation $w^4 - 8w^3 + 8w^2 + 32w - 48$

$w^4 - 8w^3 + 8w^2 + 32w - 48 = (w-2)(w-2)(w+2)(w-6)$

$D_1 = [(w+2)(w-2)(w-2) + (w-2)(w-2)(w-6) + (w+2)(w-2)(w-6) + (w+2)(w-2)(w-6)]$

$= 4w^3 - 24w^2 + 16w + 32$

$D_2 = 2![(w+2)(w-2) + (w+2)(w-2) + (w+2)(w-6) + (w-2)(w-2) + (w-2)(w-6) +(w-2)(w-6)]$

$= 12w^2 - 48w + 16$

$D_3 = 3![(w+2) + (w-2) + (w-2) + (w-6)]$

$= 24w - 48$

$D_4 = 4! = 24$

2.3.4. Differentials of fifth power of y

The Guakasamuccaya Sūtra can be utilized to solve the given equation, where the variable's greatest power is of the fifth order. This involves factoring the expression into 5 subparts as follows:

$t_1 = (w+m)$, $t_2 = (w+n)$, $t_3 = (w+p)$, $t_4 = (w+q)$, $t_5 = (w+r)$

D_1= 1! [Sum of factor product taken (5-1)=4 at a time]

$D_1 = t_1t_2t_3t_4 + t_1t_2t_3t_5 + t_1t_2t_4t_5 + t_1t_3t_4t_5 + t_2t_3t_4t_5$

$= (w+m)(w+n)(w+p)(w+q)+(w+m)(w+n)(w+p)(w+r) + (w+m)(w+n)(w+q)(w+r) +$

$(w+m)(w+p)(w+q)(w+r)+(w+n)(w+p)(w+q)(w+r)$

D_2= 2! [Sum of factor product taken (5-2)=3 at a time]

$D_2 = 2!\ [(w+m)(w+n)(w+p) + (w+m)(w+n)(w+q) + (w+m)(w+n)(w+r) + (w+n)(w+p)(w+q) +(w+n)(w+p)(w+r)]$

D_3= 3! [Sum of factor product taken (5-3)=2 at a time]

$D_3 = 3!\ [t_1t_2 + t_1t_3 + t_1t_4 + t_1t_5 + t_2t_3 + t_2t_4 + t_2t_5 + t_3t_4 + t_3t_5 + t_4t_5]$

$= 3!\ [(w+m)(w+n)+(w+m)(w+p)+(w+m)(w+q)+(w+m)(w+r)+(w+n)(w+p) +$

$(w+n)(w+r)+\ldots\ldots]$

D_4= 4![Sum of factor product taken (5-4)=1 at a time]

D_4= 4! [t_1 + t_2 + t_3 +t_4 + t_5]

D_4= 24 [(w+m)+(w+n)+(w+p)+(w+q)]

D_5 = 5! = 120

Example: Find differentials of expression

$w^5 + 20w^4 + 155w^3 + 580w^2 + 1044w + 720$

Factors are (w+2), (w+3), (w+4), (w+5) and (w+6)

D_1= [(w+2)(w+3)(w+4)(w+5)+(w+2)(w+3)(w+4)(w+6)+(w+2)(w+3)(w+5)(w+6)+

(w+2)(w+4)(w+5)(w+6)+(w+3)(w+4)(w+5)(w+6)]

$= 5w^4 + 80w^3 + 465w^2 + 1160w + 1044$

D_2= 2![(w+2)(w+3)(w+4)+(w+2)(w+3)(w+5)+(w+2)(w+3)(w+6) +

(w+3)(w+4)(w+5)+(w+3)(w+4)(w+6)+……..]

$= 20w^3 + 240w^2 + 930w + 1160$

D_3= 3![(w+2)(w+3) + (w+2)(w+4)+(w+2)(w+5)+(w+2)(w+6) +

(w+3)(w+4)+(w+3)(w+5)+ (w+3)(w+6)+…..]

$= 60w^2 + 480w + 930$

D_4= 4! [(w+2)+(w+3)+(w+4)+(w+5)+(w+6)]

$= 24\ [\ 5e + 20] = 120w + 480$

D_5 = 5! = 120

3. INTEGRAL CALCULUS

Integral Calculus is an important aspect of calculus which is inverse process of differentiation. It helps in finding the anti-derivatives of a function.

3.1. Using Urdhva Triyagbhyam Sutra to integrate the product of two functions

Urdhva Triyagbhyam Sutra can be effectively used in finding the integral of a product of functions.

Example: Evaluate $\int w^5 e^w$

Vedic Method

Step 1: Write the two functions one below the other.

w^5

e^w

Step 2: Differentiate the first function and put its value adjacent to it, to obtain the final result, differentiate it as many times until the differentiated result is zero.

$w^5 \quad 5w^4 \quad 20w^3 \quad 60w^2 \quad 120w \quad 120$

e^w

Step 3: Integrate the second functions as many times we differentiated the previous value and keep writing its value adjacent to it.

$w^5 \quad 5w^4 \quad 20w^3 \quad 60w^2 \quad 120w \quad 120 \quad 0$

$e^w \quad e^w \quad e^w \quad e^w \quad e^w \quad e^w \quad e^w$

Step 4: Apply Urdhva Triyagbhyam sutra. The product of the integrated and differentiating value is the subject of the vertical operation. The vertical operation will be used if the term under integration can be easily integrated, otherwise, the first term will be successively differentiated and the second term will be successively integrated.

$w^5 \quad 5w^4 \quad 20w^3 \quad 60w^2 \quad 120w \quad 120 \quad 0$

$e^w \quad e^w \quad e^w \quad e^w \quad e^w \quad e^w \quad e^w$

Step 5: Add the product with a consecutive + and – sign.

$$\int w^5 e^w \ = \ w^5e^w - 5w^4e^w + 20w^3e^w - 60w^2e^w + 120we^w - 120e^w$$

Conventional Method

To find $\int w^5 e^w$

For Integral of U.V form:

Formula: $U \int V - \int [(\frac{d}{dw} U) \ \int V \ ]$

Solving for $\int w^5 e^w$,

$= w^5 \ \int e^w - \ \int [(\frac{d}{dw} w^5) \ \int e^w]$

$= w^5 e^w - \int [(5w^4) \ e^w]$

$= w^5 e^w - \ 5 \int w^4 \ e^w$

Now, separately integrating $\int w^4 \ e^w$,

$= w^5 e^w - 5\left\{w^4 \ \int e^w - \ \int [(\frac{d}{dw} w^4) \ \int e^w]\right\}$

$= w^5 e^w - 5\{w^4 \ e^w - \ \int [(4w^3) \ e^w]\}$

$= w^5 e^w - 5w^4 e^w + 5 \int 4w^3 \ e^w$

$= w^5 e^w - 5w^4 e^w + 20 \int w^3 \ e^w$

Now, separately integrating $\int w^3 \ e^w$,

$= w^5 e^w - 5w^4 e^w + 20\left\{w^3 \ \int e^w - \ \int [(\frac{d}{dw} w^3) \ \int e^w]\right\}$

$= w^5e^w - 5w^4e^w + 20\{w^3e^w - \int[3w^2\ e^w\}$

$= w^5e^w - 5w^4e^w + 20w^3e^w - 60\ \int w^2e^w$

If, we keep on integrating, result we get is,

RESULT: $= w^5e^w - 5w^4e^w + 20w^3e^w \ldots\ldots\ldots\ldots$

Example: Evaluate $\int w^4Cos(w)dw$

Vedic Method

$$\begin{array}{ll} w^4 & 4w^3 \\ Cos(w) & Sin(w) \end{array}$$

$\int w^4Cos(w)dw = w^4Sin(w) - \int 4w^3Sin(w)dw$

Since, we can further integrate the term under integration, we have to successive differentiate and integrated both terms.

$$\begin{array}{lllll} w^4 & 4w^3 & 12w^2 & 24w & 24 \\ Cos(w) & Sin(w) & -\mathrm{Cos}(w) & -Sin(w) & Cos(w) \end{array}$$

$= w^4Sin(w) + 4w^3Cos(w) - 12w^2Sin(w) - 24wCos(w) + \int 24Cos(w)$

Conventional Method

$\int w^4Cos(w)dw$

Formula: $U\int V - \int[(\frac{d}{dw}U)\ \ \int V\ ]$

$= w^4 \int Cos(w)dw - \int[(\frac{d}{dw}w^4)\int Cos(w)dw]dw$

$= w^4Sin(w) - \int[(4w^3)Sin(w)]dw$

$= w^4Sin(w) - \int 4w^3Sin(w)dw$

3.2. Integration based on partial fraction by Parāvartya Yojayet Sūtra

In the case when f(x) and h(x) are two polynomials, then f(x)/h(x) is a new relationship between two polynomials whose rationality depends on the possible values for its denominator, i.e. h(x) has a non-zero value, the function is rational, and its degree exceeds f(x), making it proper. Using the following table, we can represent this relationship as partial fractions where A, B, and C are real numbers.

FORM OF RATIONAL FUNCTION	FORM OF PARTIAL FRACTION
1. $\frac{px+q}{(x-a)(x-b)}$	$\frac{A}{x-a} + \frac{B}{x-b}$

2. $\frac{px+q}{(x-a)^2}$	$\frac{A}{x-a}+\frac{B}{(x-a)^2}$
3. $\frac{px^2+qx+r}{(x-a)(x-b)(x-c)}$	$\frac{A}{x-a}+\frac{B}{(x-b)}+\frac{C}{x-c}$
4. $\frac{px^2+qx+r}{(x-a)^2(x-b)}$	$\frac{A}{x-a}+\frac{B}{(x-a)^2}+\frac{C}{x-b}$
5. $\frac{px^2+qx+r}{(x-a)(x^2+bx+c)}$	$\frac{A}{x-a}+\frac{Bx+c}{x^2+bx+c}$

Given any integral of the form of rational function, one can convert the same in partial fraction. Vedic Mathematics provides the technique to solve for the values of A and B in partial fraction so that the integral can be found.

Example: $\int \frac{1dx}{(w+1)(w+2)}$

Vedic Method

$$\int \frac{1dx}{(w+1)(w+2)} = \int \frac{A}{w+1}+\frac{B}{w+2}$$

Step 1: Put Denominator= 0

i.e., w+1=0 so, w=(-1)

w+2=0 so, w=(-2)

Step 2: The first integrable function has w+1 so omit (w+1) from original function and put the value for w as (-1), which was the result of w+1=0, and put the same in the place of A.

Similarly, the second integrable function has w+2 so omit (w+2) from original function and put the value of w as (-2) which was result of w+2=0 and put the same in place of B.

$$= \int \frac{A}{w+1}+\frac{B}{w+2}$$

$$= \int \frac{\frac{1}{-1+2}}{w+1}+\frac{\frac{1}{-2+1}}{w+2}$$

$$= \int \frac{1dw}{w+1}+\int \frac{(-1)dw}{w+2}$$

$$= \log|w+1| - \log|w+2| + c$$

Conventional Method

$$\int \frac{1dx}{(w+1)(w+2)}$$

$$\frac{1}{(w+1)(w+2)} = \frac{A}{w+1} + \frac{B}{w+2}$$

A and B are real numbers, to solve for the value of A and B,

$$\frac{1}{(w+1)(w+2)} = \frac{A(w+2)+B(w+1)}{(w+1)(w+2)}$$

$$1 = A(w+2) + B(w+1)$$

$$1 = Aw + 2A + Bw + B$$

On Equating the coefficients of equal terms, we get

A+B=0 and 2A+B=1

On Solving, A=1 and B=(-1)

Now, $\frac{1}{(w+1)(w+2)} = \frac{1}{w+1} + \frac{(-1)}{w+2}$

$$= \int \frac{1dw}{w+1} + \int \frac{(-1)dw}{w+2}$$

$$= \log|w+1| - \log|w+2| + c$$

4. CONCLUSION

Complex and time-consuming calculations can be solved more accurately and quickly using Vedic sutras than with conventional methods. The Vedic Sutras are used in differential calculus to solve derivatives and successive differentiation problems based on polynomial functions. The Urdhva Triyagbhyam Sutra and the binomial theorem can be used to solve the derivatives of product of functions and division of functions. Calculus is used to identify the roots of a quadratic problem using the Calana-Kalanabhyam sutra. The Gunaka Samuccaya sutra explains how to use a polynomial's factors to get its derivative. Comparatively, integration based on partial fraction using the Paravartya Yojayet Sutra and integration by parts using the Urdhva Triyagbhyam Sutra can be solved more quickly. Using Vedic mathematics sutras improves memory and increases mental ability.

REFERENCES

Tiratha, B.K. (1965). Vedic Mathematics, Motilal Banarasi Dass, New Delhi.

Williams, R. (2005). Vedic Mathematics, Motilal Banarasi Dass, New Delhi.

Priya, P. Goel, and A. Kumar (2021), Vedic mathematics in derivatives and integration, differential equations and partial differential equations, Journal of Mathematical Problems, Equations, and Statistics, vol 2, pp 27-32.

D. N. Garain, Sanjeev Kumar (2018), "Analytical Discussion for Existence of Idea of Calculus in Vedic and Post Vedic Periods," International Journal of Mathematics Trends and Technology (IJMTT), vol. 61, no. 2, pp. 85-88.